Engineering Notes

Bending-Torsion Transfer Matrices Associated with Swept Surfaces

KENT L. LAWRENCE*
Arlington State College, Arlington, Texas

IN an earlier paper Lin[1] discussed the advantages to be gained from using transfer matrices for calculating the steady response of flexible aircraft structures. For coupled bending-torsion of swept wing and tail surfaces it is desirable from the standpoint of the aerodynamic calculations to offset the mass elements in a streamwise direction rather than perpendicular to the elastic axis. The mass point transfer matrix for such an arrangement is derived below.

The streamwise offset mass element is shown in Fig. 1, in which the notation and positive sign conventions follow those of Pestel and Leckie.[2] Forcing functions may be applied at the elastic axis 0 (f_1, Q_1, C_1), at the mass element A (f_2, Q_2, C_2), or at some point along the streamwise strip B (f_3, Q_3, C_3), depending upon the application. If U is taken to be the downward displacement of the mass element, Γ and Δ its rotations about the axes indicated, then the equations of motion are

$$I\ddot{\Gamma} = M^R - M^L + Q_1 + Q_2 + Q_3 + (f_1 + V^R - V^L)e \sin\beta - f_3(h - e)\sin\beta \quad (1)$$

$$m\ddot{U} = V^R - V^L + f_1 + f_2 + f_3 \quad (2)$$

$$J\ddot{\Delta} = T^R - T^L + C_1 + C_2 + C_3 - (f_1 + V^R - V^L)e \cos\beta + f_3(h - e)\cos\beta \quad (3)$$

where m is the total mass, J the torsional mass moment of inertia about the x' axis, I the rotatory mass moment of inertia about the y' axis, and M, V, and T are the beam stress resultants to the left and right of the mass point. For small deformation angles, the mass point displacements and the displacements at the elastic axis are related by

$$U = w + \phi e \cos\beta - \psi e \sin\beta \qquad \Delta = \phi \qquad \Gamma = \psi \quad (4)$$

where β is the angle of sweep.

For steady harmonic motion of frequency Ω, $\ddot{w} = -\Omega^2 w$, etc., and the two sets of equations combine to give

$$M^R = M^L - \Omega^2(I + me^2\sin^2\beta)\psi + \Omega^2(me\sin\beta)w + \Omega^2(me^2\sin\beta\cos\beta)\phi + [(f_3h + f_2e)\sin\beta + (Q_1 + Q_2 + Q_3)] \quad (5)$$

$$V^R = V^L - m\Omega^2 w - \Omega^2(me\cos\beta)\phi + \Omega^2(me\sin\beta)\psi - (f_1 + f_2 + f_3) \quad (6)$$

Received March 28, 1966.

* Assistant Professor of Engineering Mechanics.

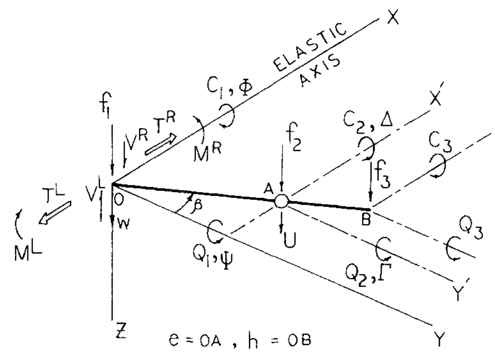

Fig. 1 Sign conventions for streamwise offset mass element.

$$T^R = T^L - \Omega^2(J + me^2\cos^2\beta)\phi - \Omega^2(me\cos\beta)w + \Omega^2(me^2\cos\beta\sin\beta)\psi - [(f_3h + f_2e)\cos\beta + (C_1 + C_2 + C_3)] \quad (7)$$

For simplification, let

$$I_{xx} = J + me^2\cos^2\beta \qquad I_{yy} = I + me^2\sin^2\beta$$
$$I_{xy} = me^2\sin\beta\cos\beta \qquad S_y = me\sin\beta$$
$$S_x = me\cos\beta \qquad \text{and}$$
$$Q' = [(f_3h + f_2e)\sin\beta - (Q_1 + Q_2 + Q_3)]$$
$$f' = -(f_1 + f_2 + f_3) \quad (8)$$
$$C' = -[(f_3h + f_2e)\cos\beta + (C_1 + C_2 + C_3)]$$

The state vector to the right of the mass element is then related to that on the left by Table 1.

Table 1 Relation of the two state vectors

$$\begin{array}{c} R \\ \begin{bmatrix} w \\ \psi \\ M \\ V \\ \phi \\ T \\ 1 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \Omega^2 S_y & -\Omega^2 I_{yy} & 1 & 0 & \Omega^2 I_{xy} & 0 & Q' \\ -\Omega^2 m & \Omega^2 S_y & 0 & 1 & -\Omega^2 S_x & 0 & f' \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -\Omega^2 S_x & \Omega^2 I_{xy} & 0 & 0 & -\Omega I_{xx} & 1 & C' \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{c} L \\ \begin{bmatrix} w \\ \psi \\ M \\ V \\ \phi \\ T \\ 1 \end{bmatrix} \end{array} \quad (9)$$

The elastic field transfer matrix for combined bending and torsion may be found in either Ref. 1 or 2.

References

[1] Lin, Y. K., "Transfer matrix representation of flexible airplanes in gust response study," J. Aircraft 2, 116–121 (1965).

[2] Pestel, E. C. and Leckie, F. A., *Matrix Methods in Elastomechanics* (McGraw-Hill Book Co. Inc., New York, 1963), p. 375.